Nuclear Multifragmentation Critical Exponents

In a recent Letter [1] the EoS collaboration presented data of fragmentation of 1 A GeV gold nuclei incident on carbon. By analyzing moments of the fragment charge distribution, the authors claim to determine the values of the critical exponents γ, β, and τ for finite nuclei. These data represent a crucial step forward in our understanding of the physics of nuclear fragmentation. However, as we will show in the following, the analysis presented in [1] is not sufficient to support the claim that the critical exponents for nuclear fragmentation have been unambiguously determined.

The main difficulty in observing critical behavior in nuclear fragmentation is that nuclei cannot be prepared and held near the temperature and density associated with the critical point. Instead, complicated nuclear reactions must be used to excite the nuclei, which may then expand to conditions sufficiently close to the critical point. The time spent at these conditions during the reaction is an open question. However, even if one assumes that conditions sufficiently close to critical are explored in some of the observed reactions, there remain at least two problems with interpreting the subsequent experimental signals. One is to identify which of the resulting particles have participated in the equilibrated system near the critical conditions, and which have resulted from the "pre-equilibrium" stage of the reaction. The other problem is to measure the "temperature" of the decaying system. These two problems are interrelated in the procedure used in Ref. [1], where the authors assume that the observed multiplicity can be used as an indicator of temperature. In this comment, we use the percolation model of nuclear fragmentation [2] to demonstrate the nature of these problems in determining critical exponents.

In percolation models one uses a bond breaking parameter p with values between 0 and 1, including p_c , the "critical value". In this model, near the critical point, the charge of the largest fragment is $Z_{\max} \propto (p-p_c)^{\beta}$, and the second moment of the mass distribution is $M_2 \propto |p-p_c|^{-\gamma}$, with $\beta = 0.41$ and $\gamma = 1.8$.

We follow the analysis of [1] and use $m_c = 26$, in accordance with the cut employed in [1]. It is worth noting that we also find roughly identical values of γ and γ' for the liquid and gas branches, if we examine $\ln M_2$ vs. $\ln|m-m_c|$ and use $m_c = 26$. However, this value of $m_c = 26$ is lower than

$m(p_c)$, and the numerically extracted value of $\gamma = \gamma'$ is approximately 1.2 to 1.4, significantly lower than 1.8, the critical exponent for the percolation model. These two observations already indicate possible problems in trying to find the critical exponents via the analysis employed in [1].

One would only expect $Z_{\max} \propto (m - m_c)^{\beta}$ and $M_2 \propto |m - m_c|^{-\gamma}$, if multiplicity and p are related in a strictly linear way. Percolation calculations show, however, that on average m rises monotonically with p, but not linearly near the critical value. Furthermore, for a given value of p, there is always a distribution in the values of m. Thus, any translation of an observable, O, in terms of p into terms of m involves a non-trivial convolution, $O(m) = \int dp\, m(p) \otimes O(p)$.

The results in Fig. 1 illustrate the difficulty in extracting the critical exponent β. Here we plot $\ln Z_{\max}$ vs. $\ln|m - m_c|$. The open circles represent the result from a percolation model with $Z = 79$ charges. 10^4 events were generated to provide statistics comparable to the experimental data. The best fit to these points results in a slope 'β' = 0.55. For comparison, the solid line has a slope of $\beta = 0.41$, the nominal value for percolation. This demonstrates that the slope extracted from the present plot is not the critical exponent β.

In addition to this problem, one expects that a portion of the observed multiplicity is comprised of pre-equilibrium particles, which are not part of the equilibrated system. In cascade simulations, the number of such charges varies from 0 to possibly 15 for the system used in Ref. [1]. If it is not possible to experimentally (via momentum space analysis, for example) remove these, then complicated modelling of the pre-equilibrium reaction dynamics is needed. To illustrate the influence of such pre-equilibrium contamination on the extraction of meaningful values for the critical exponent β we show the results of a calculation in which we assume that 10 pre-equilibrium particles exist along with an equilibrated system of 69 charges. The remaining system of 69 charges is then fragmented, again by using the percolation model. Since the total observed multiplicity includes the pre-equilibrium particles, plotting $\ln Z_{\max}$ vs. $\ln|m - m_c|$ results in the crosses in Fig. 1. For comparison, the dashed line has a slope of 'β' = 0.29. (Coincidentally, this is the value extracted from the experimental data.)

Similarly, when following the steps of analysis in [1] for determination of γ, we find significant contamination of the 'liquid' branch due to pre-equilibrium emission. In particular, we find that the extremely large offset in

$\ln|M_2|$ between the liquid and gas branch (about 3.5) in Fig. 2 of [1] seems to be caused by this contamination, in combination with the procedure of dropping the largest fragment on the "liquid" side of the critical point and keeping it on the "gas" side.

Since the analysis of [1] does not work for a simple model with known values of critical exponents, one should not expect it to yield the correct critical exponents for the data, either.

In summary, while we applaud the effort and beautiful data of [1], we do not agree with the conclusion that the critical indices of nuclear fragmentation have been determined yet.

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Fig. 1. Percolation model simulation of the fragmentation of gold. Circles: $Z = 79$ system fragmenting; crosses: $Z = 69$ system fragmenting plus 10 pre-equilibrium protons. For comparison, the solid line has a slope of $\beta = 0.41$, and the dashed line 'β' = 0.29.